

Problem 1.53

[Difficulty: 5]

1.53 A syringe pump is to dispense liquid at a flow rate of 100 mL/min. The design for the piston drive is such that the uncertainty of the piston speed is 0.001 in./min, and the cylinder bore diameter has a maximum uncertainty of 0.0005 in. Plot the uncertainty in the flow rate as a function of cylinder bore. Find the combination of piston speed and bore that minimizes the uncertainty in the flow rate.

Given: Syringe pump to deliver 100 mL/min $\delta V = 0.001 \frac{\text{in}}{\text{min}}$ $\delta D = 0.0005 \cdot \text{in}$

Find: (a) Plot uncertainty in flow rate as a function of bore.
(b) Find combination of piston speed and bore resulting in minimum uncertainty in flow rate.

Solution: We will apply uncertainty concepts.

Governing Equations: $Q = \frac{\pi}{4} \cdot D^2 \cdot V$ (Flow rate in syringe pump)

$$u_R = \pm \left[\left(\frac{x_1}{R} \frac{\partial R}{\partial x_1} u_{x_1} \right)^2 + \dots \right]^{\frac{1}{2}} \quad (\text{Propagation of Uncertainties})$$

Now solving for the piston speed in terms of the bore: $V(D) = \frac{4 \cdot Q}{\pi \cdot D^2}$

So the uncertainty in the flow rate is: $u_Q = \pm \left[\left(\frac{D}{Q} \frac{\partial Q}{\partial D} u_D \right)^2 + \left(\frac{V}{Q} \frac{\partial Q}{\partial V} u_V \right)^2 \right]^{\frac{1}{2}} = \pm \left[\left(\frac{D}{Q} \frac{2Q}{D} u_D \right)^2 + \left(\frac{V}{Q} \frac{Q}{V} u_V \right)^2 \right]^{\frac{1}{2}}$

$u_Q = \pm \left[(2u_D)^2 + (u_V)^2 \right]^{\frac{1}{2}}$ where $u_D = \frac{\delta D}{D}$ $u_V = \frac{\delta V}{V}$ The uncertainty is minimized when $\frac{\partial u_Q}{\partial D} = 0$

Substituting expressions in terms of bore we get: $D_{\text{opt}} = \left[\frac{32}{\pi^2} \cdot \left(\frac{\delta D \cdot Q}{\delta V} \right)^2 \right]^{\frac{1}{6}}$

Substituting all known values yields $D_{\text{opt}} = 1.76 \cdot \text{in}$

Plugging this into the expression for the piston speed yields $V_{\text{opt}} = 2.50 \cdot \frac{\text{in}}{\text{min}}$ and the uncertainty is $u_{\text{opt}} = 0.0694 \cdot \%$

Graphs of the piston speed and the uncertainty in the flowrate as a function of the bore are shown on the following page.

